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Manuscript received May 2, 1961; revision received October 24, 1961; paper accepted October 27, 1961. Paper presented at A.I.Ch.E. Cleveland meeting.

Transport Characteristics of Suspensions: Part IV. Friction Loss of Concentrated-Flocculated Suspensions in Turbulent Flow

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Turbulent flow friction factors were determined for flocculated suspensions of thoria, kaolin, and titania in tubes ⅛- to 1-in. diameter. The non-Newtonian laminar flow data were arbitrarily fitted with the Bingham plastic model. With this model the range of yield stress values was 0.018 to 1.39 $lb._f$/sq. ft., with a maximum ratio of coefficient of rigidity to viscosity of suspending medium of 11.1. The volume fraction solids were varied from 0.042 to 0.23.

Two types of behavior were observed depending on the value of the yield stress. For yield values less than 0.5 $lb._f$/sq. ft. the turbulent friction factors were always less than those for Newtonian fluids but tended to approach the Newtonian values as the Reynolds number was increased. For yield values greater than 0.5 $lb._f$/sq. ft. the friction factors were again less than those for Newtonian fluids but tended to diverge from the Newtonian values as the Reynolds number was increased. Both sets of data were correlated with the Blasius relation with the coefficient and exponent given in terms of the laminar flow properties and the volume fraction solids.

The mechanics of the flow of solids suspended in liquids has attracted increased attention during the last decade, encompassing not only basic research but also applied studies directed toward large-scale operations, such as the transport of coal in commercial pipelines (*1*), and more unusual applications such as high energy solid additives to jet fuels (*2*) and the transport of fertile material in homogeneous nuclear reactors (*3*). An important aspect of the applied problems is the friction loss characteristics of the suspensions in turbulent flow. Although the primary measurements required for the determination of the friction loss characteristics (that is the pressure drop as a function of flow rate) are relatively simple to make, the subsequent interpretation and correlation are complicated by several factors. Not the least of these is that friction-loss correlations for Newtonian fluids are primarily empirical in nature. Although it has been established experimentally for Newtonian fluids that Reynolds number similarity gives similarity of friction factors, no equivalent solutions of the exact equations of motion are available. This means that in order to compare the data on the characteristics of suspension friction loss with data for ordinary fluids, great care must be exercised in the selection of the Reynolds number in order to insure similarity. The ambiguity in the determination of a suitable Reynolds number for suspensions arises because of the effect of the solid phase on the laminar viscosity. Particle size and shape, as well as solids concentration, are the principal factors affecting the viscosity. Suspensions of large symmetrically shaped particles (approximately 50 μ or larger) have Newtonian flow characteristics with viscosity a function of volume fraction solids (*4*). Suspensions of smaller particles or of assymetrically shaped particles (platelike or needlelike) possess non-Newtonian flow characteristics with apparent viscosities a function of the rate of shear, as well as particle size, shape and concentration (*5*). Thus possible viscosities which must be considered for use in suspension friction loss correlations include:

1. The viscosity of the suspending medium.
2. The suspension viscosity, a function only of solids concentration for Newtonian suspensions.
3. The apparent viscosity, a function of both shear rate and solids size, shape and concentration for non-Newtonian suspensions.
4. The limiting viscosity at high rates of shear, a function only of solids

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characteristics for non-Newtonian suspensions.

Since there is no a priori basis for selecting one laminar-suspension viscosity to represent the viscosity term in the Reynolds number, recourse must be made to experiment. Maude and Whitmore (*6*) studied the friction-loss characteristics of symmetrically shaped, large particle size Newtonian suspensions (particle size ¼ to 2 times the thickness of the laminar sublayer) and used the viscosity of the suspending medium in their calculation of the Reynolds number. On the other hand, in studies with non-Newtonian suspensions of symmetrically shaped particles which were 0.05 to 0.006 the thickness of the laminar sublayer, the limiting viscosity at high rates of shear was shown to be the most suitable viscosity to use in the calculation of the Reynolds number (*7*).

The objective of this study was to measure the friction-loss characteristics of extremely non-Newtonian suspensions of symmetrically shaped particles and determine the correct viscosity to give Reynolds number similarity. Once a suitable Reynolds number was demonstrated, the objective was to generalize the friction-loss characteristics in terms of the laminar flow properties and qualitatively compare the results with those obtained with Newtonian fluids in order to determine the general mechanics involved in the turbulent flow of suspensions.

BASIC CONCEPTS

Non-Newtonian Fluids

Non-Newtonian characteristics of fluids may have basically different origins (*5, 8, 9*); for example they may arise from organic polymer chains in solution, assymetric shape of the particles in suspension, or colloidal forces between small particles in suspension. Many fluids belonging to all three of the above categories have the common property of becoming less viscous as the shear rate is increased and have laminar flow shear diagrams which are almost indistinguishable. However, as emphasized by Metzner (*10*), the turbulent flow behavior of fluids in the different categories may be markedly different. Extensive studies have been presented for materials in the first two categories given above (*10, 11*); the present paper is concerned with suspensions belonging to the third category, with the further restriction of symmetrical particle shape.

The effect of particle size, shape, and concentration on the laminar flow physical properties of such suspensions was presented in a previous paper (*12*). The data were interpreted in terms of the well-known Bingham plastic (*13*) model:

$$du/dr = g_c(\tau - \tau_y)/\eta \text{ for } \tau \geqq \tau_y \tag{1a}$$

$$du/dr = 0 \qquad \text{for } \tau < \tau_y \tag{1b}$$

in which the yield stress and the coefficient of rigidity (or equivalently the limiting viscosity at high rates of shear) are the physical properties which characterize the suspension. The parameters τ_y and η were related to the solid phase properties by the expressions

$$\tau_y = F\Psi_1\varphi^3/D_p^2 \tag{2a}$$

$$\eta/\mu = \exp(2.5 + 14\Psi_2/\sqrt{D_p})\varphi \tag{2b}$$

The Bingham plastic model was selected for the laminar flow properties correlation because it had been shown to be particularly suitable for correlating turbulent flow heat transfer and friction-loss data (*7*). The advantages of this model for turbulent flow correlations were:

1. It fits the data sufficiently well at high-shear rates to allow accurate effective viscosities and laminar velocity gradients at the wall to be calculated from the experimentally determined flow parameters.
2. It extrapolates to give a limiting value of the viscosity at high rates of shear that is always greater than the viscosity of the suspending medium.

With these coefficients used, the turbulent friction factors for suspensions with yield values less than 0.5 lb.$_f$/sq.ft. were given by (*7*)

$$f = 0.079(\mu/\eta)^{0.48}(DV\rho/\eta)^{0.25(\mu/\eta)^{0.15}} \tag{3}$$

An important question in the flow of suspensions is whether a layer of pure suspending medium occurs at the tube wall. In one particular laminar flow study effects were observed which appeared to be a function of the ratio of particle diameter to tube diameter and which were attributed to a layer of pure fluid adjacent to the wall of the tube (*14*). If this wall layer persists in turbulent flow, it could significantly alter the friction-loss characteristics, since turbulent energy production and dissipation is confined largely to regions near the wall (*15*). Oldroyd (*16*) has examined the effect of such a layer on the experimental data and has shown that a sufficient condition for the proof that the wall layer makes a negligible contribution is simply that data taken with tubes of different diameter be in good agreement. Application of this criterion to data for suspensions of 1- to 3-μ particles flowing in tubes of ⅛- to 1-in. diameter showed no evidence of a wall layer in either laminar or turbulent flow (*7*).

EXPERIMENTAL RESULTS

Laminar and turbulent flow pressure-drop measurements were made in tubes with nominal diameters of ⅛-, ⅜-, and 1-in. with the same equipment and technique described previously (*7*). As in that study, the rheological properties of the suspensions were determined from data obtained with a viscometer tube having a nominal diameter of ⅛ in. and L/D ratio of 1,000. The choice of this tube diameter was dictated by pronounced wall effects observed with micron-sized suspensions when tubes of smaller diameters are used (*12*). Although the use of tubes of ⅛-in. diameter limited the maximum shear stress for which laminar data could be obtained (because of the onset of turbulence), this was believed to be more desirable than the use of high-shear rate data from smaller tubes which contained a wall effect of unknown magnitude.

The present study included data for suspensions of particles of kaolin and titanium dioxide as well as thorium oxide in order to confirm the generality of the previous correlations, and in addition the range of the data was extended to include suspensions having larger values of the yield stress. This extension disclosed an entirely different trend of friction factor with increasing non-Newtonian behavior, thus necessitating a more complex expression for the friction factor than had been presented for suspensions having yield values less than 0.5 lb.$_f$/sq.ft. [Equation (3)]. Before proceeding with the discussion of this phenomena, it is necessary to establish that the choice of the viscosity term made in the previous paper (*7*) remains valid for the more non-Newtonian suspensions treated in the present paper.

The criterion for the selection of a viscosity term was that there should be good agreement of turbulent friction factors as a function of Reynolds number for any given suspension, independent of tube diameter and velocity. The limiting viscosity at high rates of shear η, a constant for each suspension, gave such similarity in the previous study (*7*). Although the viscosity of the suspending medium would also have given similarity, η was chosen because the particle size of the solid phase was always much smaller than the thickness of the laminar sublayer, calculated from the expression $\delta = 16.4\ D/N_{Re}\sqrt{f}$, ($D_p = 1.5$ to $3\ \mu$, $\delta = 60$ to $500\ \mu$); hence suspension was being sheared even in the laminar sublayer. Application of the above criterion for friction factor similarity to the present high yield stress data ($\tau_y > 0.5$ lb.$_f$/sq.ft., $D_p = 0.4$ to $2.9\ \mu$, $\delta = 65$ to $230\ \mu$) gave results similar to those for the low yield stress materials. This is shown in Figure 1 for suspensions of thorium oxide having yield values of 0.69 and 1.25 lb.$_f$/sq.ft. The coordinates of the diagram on the left permit the correlation of laminar flow data by a single line, with the turbulent flow data for different tube diameters branching off on different lines. The good agreement of the laminar flow data, independent of the tube diameter, insured that wall effects (which may become important as the ratio of particle diameter to tube diameter is increased) were negligible for the systems studied, and therefore the coefficient of rigidity and yield stress were unique properties of the suspension. On the right side of Figure 1 these same data are replotted as f vs. N_{Re}, coordinates which, with the proper selection of the viscosity, permit the correlation of the turbulent flow data by a single line. [With these coordinates the laminar flow data for different tube diameters are shown on different lines whose positions are determined by the value of the yield stress, the coefficient of rigidity, the suspension concentration, and the tube diameter (*17*).] The good agreement of the turbulent flow data show that there was no detectable wall effects for tubes from ⅛ to 1 in. in diameter and that friction factor similarity with high yield stress suspensions was obtained with the limiting viscosity at high rates of shear η in the calculations of the Reynolds number. This definition of Reynolds number was used in the subsequent analysis of the data.

Turbulent friction factors for non-Newtonian suspensions were always below those for Newtonian fluids; however two entirely different trends with increasing Reynolds number were observed depending on the value of the non-Newtonian properties. For yield values less than 0.5 lb.$_f$/sq.ft. the suspension friction factors tended to approach those for Newtonian fluids as the Reynolds number was increased. In contrast to this the friction factors for suspensions with yield values greater than 0.5 lb.$_f$/sq.ft. tended to diverge from the Newtonian values as the Reynolds number was increased. The extent of the change in behavior is illustrated in Figure 2, which shows data for the Blasius coefficient B and exponent b plotted as a function of yield stress. The values of B and b were determined by fitting the friction factor line for each suspension with the expression (*18*)

$$f = B\ N_{Re}^{-b} \tag{4}$$

The data for twenty four different suspensions* are summarized in Figure 2, and the range of solids characteristics and operating conditions are given in Table 1. In all cases the smaller value of the maximum Reynolds number shown in the table was observed with the most non-Newtonian suspensions. The choice of yield stress as the independent variable in Figure 2 was made only to indicate the trend and does not imply that this was the only variable of importance. (Choice of η/μ as the independent variable also indicated two different types of behavior, although the shape of the curves was somewhat different.)

Table 1. Solids Characteristics and Range of Operating Conditions in Present Study

Material	Particle size on weight basis Diam., μ	σ	Conc. range, vol. fr. solids	η/μ range	Yield stress range, lb.$_f$/sq. ft.	Maximum Reynolds no. $\frac{DV\rho}{\eta} \times 10^{-4}$
ThO_2-I (12)	1.05	2.3	0.075-0.130	3.4-10	0.018-0.59	4.9-13.0
ThO_2-II	0.60	3.0	0.042-0.130	2.9-10	0.024-1.39	6.6-14.0
ThO_2-III	0.74	2.2	0.055-0.16	2.4-11	0.024-1.25	4.8-14.4
Kaolin	2.85	4.0	0.076-0.25	3.3-9.2	0.045-0.76	3.8-9.6
TiO_2	0.40	1.6	0.036-0.079	3.3-9.3	0.12-1.13	0.4-1.4

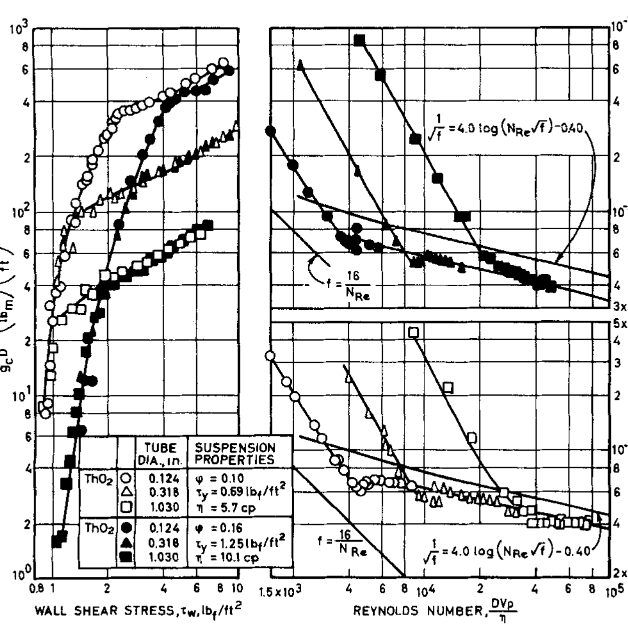

Fig. 1. Pseudo shear diagram and Fanning friction factor plot for concentrated suspensions showing agreement of laminar and turbulent data, respectively, as tube diameter was varied.

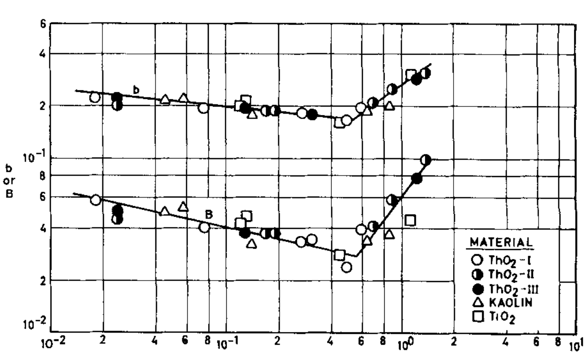

Fig. 2. Effect of yield stress on Blasius coefficients relating the Reynolds number and the Fanning friction factor.

* An appendix containing laminar flow properties and turbulent friction loss parameters has been deposited as document 7079 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or for 35-mm. microfilm.

TABLE 2. RESULTS OF LEAST-SQUARES ANALYSIS OF BLASIUS COEFFICIENT AND EXPONENT IN TERMS OF NON-NEWTONIAN FLOW PARAMETERS OF EQUATIONS (5a) and (5b)

	Case I	Case II (recommended values)
1. Coefficient, B		
a	0.40	0.48
d	1.91	2
x, ft.	6.16×10^{-6}	6.44×10^{-6}
Mean deviation of calculated and experimental values	−16%, +6%	−8%, +12%
2. Exponent, b		
c	0.13	0.15
e	2.03	2
x, ft.	5.55×10^{-6}	5.68×10^{-6}
Mean deviation of calculated and experimental values	−9%, +7%	−9%, +4%

CORRELATION OF RESULTS

Although Figure 2 summarizes all the experimental results, generalization requires a correlation in terms of the physical properties of the suspension and the suspending medium. Dimensional analysis shows that, aside from the Reynolds number, two different combinations of suspension physical properties are possible, η/μ and $g_c\rho\,\tau_y x^2/\mu^2$. These two parameters must be combined additively in order to represent the change of slope shown in Figure 2. When one takes into account the results for low yield stress suspensions, [Equation (3)], the expression for the coefficient and exponent of the Blasius equation, [Equation (4)], for the entire range of yield values becomes

$$B = 0.079\ (\theta^a + \Phi^d) \qquad (5a)$$

$$b = 0.25\ (\theta^c + \Phi^e) \qquad (5b)$$

where $\theta = \mu/\eta$ and $\Phi = g_c\rho\,\tau_y x^2/\mu^2$. When the yield stress is zero and η and μ are equal, Equations (5a) and (5b) reduce to the commonly accepted values for Newtonian fluids. The length term x should be of the order of the scale of eddies for which viscous forces are important, since this is the distance over which the yield stress might be expected to exert an influence on the structure of turbulent flow.

The constants and exponents of Equation (5) were determined with a nonlinear least-squares procedure. Two different cases were studied; in the first case (I) the least-squares value for the constants and exponents were determined for the present data; in the second case (II) the exponents a and c were taken to be the same as those of the previous study (7), and the exponents d and e were both assumed to be equal to 2. The results of the analysis are shown in Table 2, together with the mean deviation of the ratio of the calculated and the experimental values of B and b. The Case II values are recommended for use in Equations (5a) and (5b), since the mean deviations were somewhat less than those observed with Case I. Values of the Blasius coefficient and exponent calculated with the Case II constants are compared with experimental values in Figure 3, and although the values for the exponent b are fitted somewhat better than the values for the coefficient B, in both cases the agreement is quite satisfactory.

A rather remarkable fact disclosed by the least-squares analysis is that the length terms, given in Table 2, which were derived from both the exponent and the coefficient of the Blasius equation have essentially the same value of about 6×10^{-6} ft. This value is of the same order of magnitude as the ratio of ν/u_* for the suspending medium (for example for a viscosity of 0.9 centipoise and a density of 1.0 g./cc. the value of ν/u_* is 6.0×10^{-6} ft. when $\tau_w = 7.5$ lb.$_f$/sq.ft. and the suspension density is 1.5 g./cc.). The significance of this will be discussed in more detail in the following section, although it may be noted that on the assumption of local isotropy, $u_* = u'$ and ν/u' then provides a rough estimate of the scale of the smallest eddies (21).

PHENOMENOLOGICAL ANALYSIS

Although the dimensional analysis and data correlations were necessary for the application of the results to other systems, they provide little insight into the mechanism of the turbulent flow of non-Newtonian suspensions. Nevertheless it is clear from the form of Figure 2 and of Equations (5a) and (5b) that two separate phenomena were involved. The first was observed with suspensions having moderately non-Newtonian characteristics and was exemplified by friction factors below the Newtonian values, with the difference between the two values decreasing as the Reynolds number was increased. The second was observed for suspensions having large values of the yield stress and was exemplified by a divergence from turbulent Newtonian behavior toward a more laminarlike behavior. Information on the nature of the two different processes can be deduced by deriving an expression for the turbulent non-Newtonian friction law following the procedure used by Prandtl in deriving the universal law of friction for Newtonian fluids flowing in smooth pipes (18, 19). The key assumptions in Prandtl's hypothesis were that in the vicinity of the wall the mixing length was proportional to the distance from the wall, and that the thickness of the laminar sublayer was proportional to the ratio of the kinematic viscosity to the friction velocity. The first assumption appears equally valid for the present study because the suspensions were composed of symmetrically shaped particles which would be essentially isotropic toward turbulent fluctuations in the high-shear region in the vicinity of the wall; in addition the large shear stresses at the wall would minimize nonlinear effects due to the yield stress. In the second assumption the only question is the correct value for the kinematic viscosity for the non-Newtonian suspensions. In the previous section of this paper the limiting viscosity at high rates of shear was shown to be the correct value to give Reynolds number similarity for the same suspension flowing in tubes of different diameter. Since Prandtl's second assumption refers to the wall region where high-shear processes are of primary importance, the limiting viscosity at high rates of shear will be used in calculating the kinematic viscosity. On this basis the derivation is straightforward and gives Equations (6), (7), and (8) in which the Reynolds number is calculated with the limiting viscosity at high rates of shear. The coefficients A and C are to be related to the experimental data:

$$1/\sqrt{f} = A \log (N_{Re}\sqrt{f}) + C \qquad (6)$$

where

$$A = 2.303/\kappa\sqrt{2} \qquad (7)$$

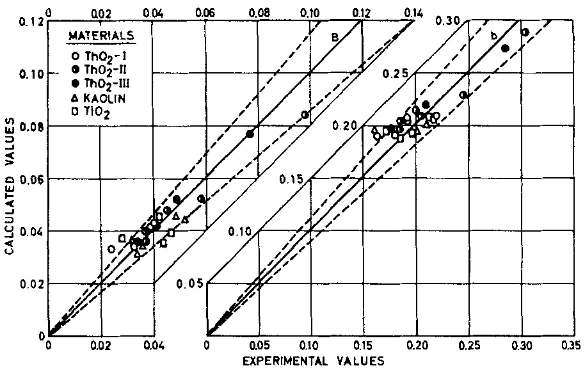

Fig. 3. Comparison of calculated and experimental Blasius coefficients.

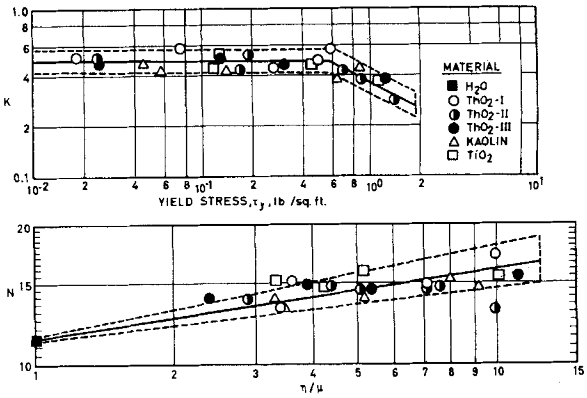

Fig. 4. Effect of non-Newtonian characteristics on dimensionless thickness of laminar wall region N and on the von Karman coefficient κ.

and

$$C = \frac{N - (2.303 \log N)/\kappa - 3/2}{2} + \frac{2.303}{\sqrt{2}} \log 1/2\sqrt{2} \qquad (8)$$

The two terms of interest are $N = u_{*w}\,\delta/\nu$, the dimensionless thickness of the layer separating the wall from the inner turbulent region, and the von Karman coefficient κ.

The first fact that can be deduced by application of Prandtl's equations is that the thickness of the layer separating the wall from the turbulent core increases continuously with increasing non-Newtonian character, regardless of the trend of values of the Blasius coefficients. This is shown in Figure 4 in which values of the wall thickness layer N calculated from Equations (6) and (8) for all the high and the low yield stress suspensions of the present study are shown as a function of η/μ. The relation between N and the suspension characteristics is given by

$$N = 11.4\ (\eta/\mu)^{0.15} \qquad (9)$$

which reduces to the commonly accepted value for Newtonian fluids when $\eta = \mu$. Such a thickening of the wall layer has been proven experimentally (*11, 22*) in two different studies. In the first (*11*) the thickening was detected by dye injection into non-Newtonian polymer solutions; however the results may not apply to the present study owing to viscoelastic effects associated with the polymer solution. The second study (*22*) also was made with polymer solutions, but since the friction-loss data agreed well with the correlation presented by Dodge and Metzner (*10*) for nonviscoelastic materials, presumably there were no viscoelastic effects. In this second study both velocity profile and heat transfer measurements indicated a definite thickening of the laminar sublayer; however no attempt was made to evaluate the thickneing quantitatively.

One possible explanation of the gradual thickening may be obtained by comparison with the results of detailed studies of Newtonian fluids. These studies showed that the production of turbulence was at a maximum very close to the wall, at a distance $y^+ \approx 11.4 \equiv N$, that is the value given by Equation (9). However the energy of turbulence was not completely dissipated where it was produced, but there was a displacement toward the axis due to diffusion in the direction of decreasing turbulent intensity (*15*). Now, flocculated solids in suspension would be expected to markedly suppress the turbulent intensity, particularly in the vicinity of the tube axis where the shear stress is less than the yield stress. When one follows the same arguments advanced for Newtonian fluids, damping of the turbulent fluctuations on the tube axis due to the flocculated solids results in an additional displacement of the maximum in the turbulence dissipation and production away from the wall, and consequently a thicker wall layer, as given by Equation (9). This explanation is also consistent with the observation that friction factors of low yield stress suspensions converge toward Newtonian friction factors as the Reynolds number is increased. That is the increase in Reynolds number is accompanied by an increase in the wall shear stress, hence by decreasing values of τ_y/τ_w, thus weakening the strength of the sink and consequently causing less deviation of the wall-layer thickness from the values expected for Newtonian fluids, and in general a behavior more characteristic of Newtonian fluids.

In contrast to the behavior of low yield stress suspensions the friction factors of high yield stress suspensions diverged from the Newtonian fluid line. This suggests that turbulent fluctuations were damped throughout the tube cross section by the suspended solids and that the flow was becoming more laminar in nature. This is supported by the effect of yield stress on the von Karman coefficient κ shown in Figure 4. The value of κ was substantially constant up to a yield stress of 0.5 lb.$_f$/sq.ft. and decreased thereafter. Since κ is commonly considered to be a measure of the average intensity of turbulent fluctuation (*15*), the decrease in κ means that the turbulent intensity was damped for suspensions with yield values greater than 0.5 lb.$_f$/sq.ft. Similar deductions have been reported by Vanoni (*20*) in studies of Newtonian suspensions; in addition he observed that the effect increased with increasing concentration and decreasing particle size.

An alternate explanation of the effects observed with high yield stress suspensions can be realized by examining the term $\Phi = g_c\rho\tau_y x^2/\mu^2$, [Equation (5)] which was controlling for large values of the yield stress. As pointed out above, the length term x is of the order of ν/u_*. [Since u_* is experimentally proportional to u', the fluctuating component of the mean velocity (*15*), then x also is the order of the scale of eddies for which viscous forces are important (*21*); that is x is the distance over which the yield stress might be expected to exert an influence on the structure of turbulent flow.] Replacing x by ν/u_* and τ_y by Equation (2*a*) one gets

$$\Phi = \varphi^3\ [(F/D_p^2)/(\rho u_*^2/g_c)] \qquad (10)$$

On this basis Φ is proportional to the volume fraction solids cubed and the ratio of the attractive forces between particles to the disruptive forces due to turbulent fluctuations. Hence the relative importance of the turbulent fluctuations at any given flow rate is diminished by the addition of more solids or by the increase of the attractive force term by a reduction in particle size. Finally, it should be noted that the quantitative relation given by Equation (10) is in agreement with Vanoni's qualitative observation cited above.

DESIGN PROCEDURE

The strong dependence on yield stress of the Φ term in Equations (5*a*) and (5*b*) permits a considerable simplification of the design procedures for low yield stress suspensions. That is for yield values less than about 0.3 lb.$_f$/sq.ft. the Φ term contributes less than 5% to the final value of the Blasius coefficient and exponent. This means that a design plot of friction factor vs. Reynolds number can be prepared with only η/μ as a parameter provided the yield stress is less than 0.3 lb.$_f$/sq.ft. However for larger values of the yield stress the complete expressions for B and b must be used. Although this means a separate line for every different set of suspension characteristics, the line can be established for any particular suspension by calculating values of friction factor for two different Reynolds numbers. Once the line has been determined, the procedure for its use is no different than with Newtonian fluids.

CONCLUSIONS

The turbulent flow of suspensions of symmetrically shaped particles possessing non-Newtonian laminar flow properties is characterized by two different types of behavior depending on the magnitude of the non-Newtonian parameters, which, for this study, were taken to be the Bingham plastic coefficients τ_y and η.

For suspensions having values of τ_y less than 0.5 lb.$_f$/sq.ft. the turbulent friction factors were less than, but tended to approach, those for Newtonian fluids as the Reynolds number was increased. The departure from Newtonian friction factors is believed to be the result of a laminar wall layer which was thickened by the non-Newtonian characteristics with the effect becoming negligible for small values of yield stress or large values of the wall shear stress, that is small τ_y/τ_w.

For suspensions having values of τ_y greater than 0.5 lb.$_f$/sq.ft. the turbulent friction factors were less than those for Newtonian fluids and tended to diverge from the values for Newtonian fluids as the Reynolds number was increased. This is believed to be due to a damping of the turbulent fluctuations by the flocculated solids, with the effect increasing both with the volume fraction solids and with the ratio of the attractive force between particles to the disruptive force due to turbulent fluctuations.

Confirmation of the above hypotheses await detailed studies of the effect of suspended solids on the velocity profile and on turbulent fluctuations. Such studies would permit the identification of the basic mechanism responsible for the observed flow phenomena, that is whether the above effects are due to suspended solids alone, the interation of the solids particles resulting in non-Newtonian flow characteristics, or to an entirely different flow phenomena or to a combination of all three effects.

ACKNOWLEDGMENT

The data were obtained at the Oak Ridge National Laboratory, and the paper was written during the tenure as a National Science Foundation Senior Postdoctoral Fellow at Cambridge University. Sincere thanks are expressed for the opportunity thus afforded.

NOTATION

A, B, C = coefficients, dimensionless
a, b, c, d, e = exponents, dimensionless
D = tube diameter, ft.
D_p = particle diameter, μ
F = force constant, 2.27×10^{-9} lb.$_f$
f = Fanning friction factor, $\tau_w/(\rho V^2/2g_c)$, dimensionless
g_c = conversion factor, (lb.$_m$/lb.$_f$) (ft./sec.^{2})
N = wall-layer thickness, Equation (8), dimensionless
N_{Re} = Reynolds number, $DV\rho/\mu$ or $DV\rho/\eta$, dimensionless
$\Delta p/L$ = pressure gradient, lb.$_f$/cu.ft.
du/dy = velocity gradient, sec.$^{-1}$
u_* = friction velocity, $\sqrt{g_c\tau/\rho}$, ft./sec.
u_{*w} = wall-friction velocity, $\sqrt{g_c\tau_w/\rho}$, ft./sec
u' = root mean square fluctuating velocity, ft./sec.
V = mean velocity, ft./sec.
x = length, [Equation (5*a*), (5*b*)], ft.
y = distance from tube wall, ft.

Greek Letters

δ = wall-layer thickness, ft.
θ = μ/η, dimensionless
κ = von Karman coefficient, dimensionless
η = coefficient of rigidity, lb.$_m$/ft. sec.
μ = fluid viscosity, lb.$_m$/ft. sec.
ν = kinematic viscosity, μ/ρ, sq. ft./sec.
ρ = density, lb.$_m$/cu.ft.
τ = shear stress, lb.$_f$/sq.ft.
τ_w = wall shear stress, $(D\Delta p/4L)$ lb.$_f$/sq.ft.
τ_y = yield stress, lb.$_f$/sq.ft.
φ = volume fraction solids, dimensionless
Φ = $g_c\rho\tau_y x^2/\mu^2$
Ψ, Ψ_1 = shape factors equal to unity for symmetrically shaped particles

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Manuscript received May 29, 1961; revision received September 28, 1961; paper accepted October 5, 1961.